



RESEARCH  
INSTITUTE.

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# SELF-REPRODUCING COINS AS UNIVERSAL TURING MACHINE



# OVERVIEW

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- Can we do it in UTXO model?
- If so, what is the practical use of it?

## **OUTLINE**

- **TURING COMPLETENESS IN BLOCKCHAIN ENVIRONMENT**
- **SCRIPTING LANGUAGE PREREQUISITES, UTXO'S, ERGO**
- **GENERAL CONSTRUCTION + PROOF OF TURING COMPLETENESS**
- **PRACTICAL CASES**
- **CONCLUSIONS**

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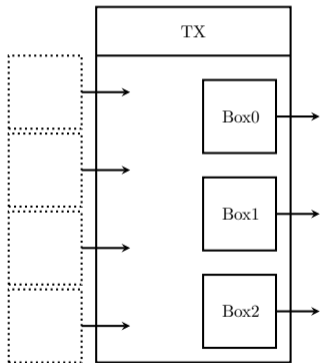
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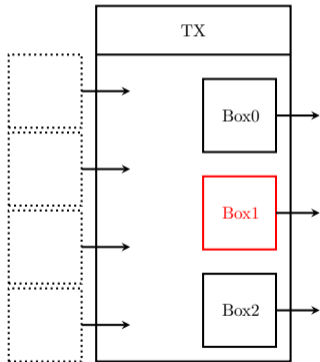
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- But imagine Ethereum<sup>⊖</sup> where there is gas limit per block, and all the state changes are reversed after transaction script execution (and payment amount is not dependent on the program). Then the Ethereum<sup>⊖</sup> is not Turing-complete.

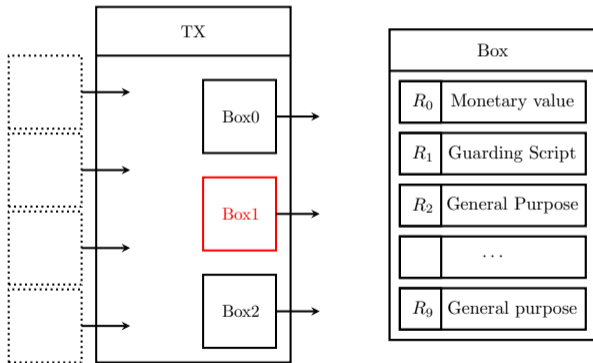
# BITCOIN<sup>⊕</sup>: BOXES, REGISTERS, SCRIPTING



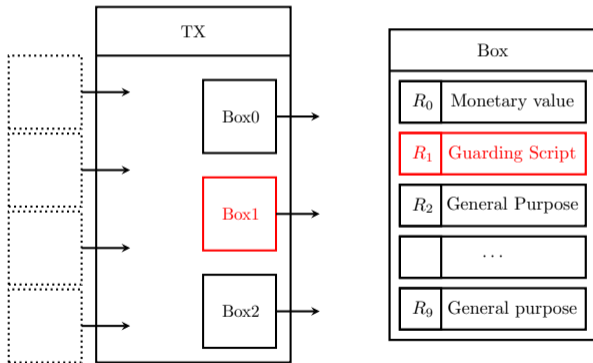
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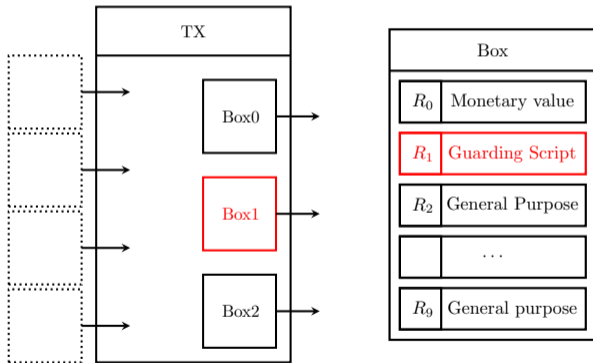


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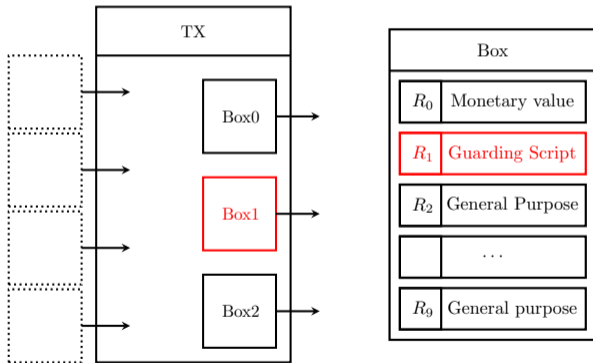
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## SCRIPTING LANGUAGE





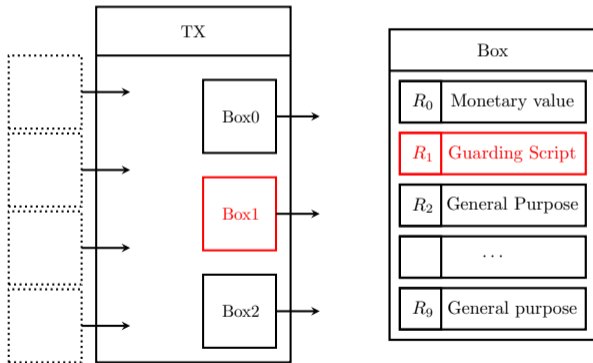
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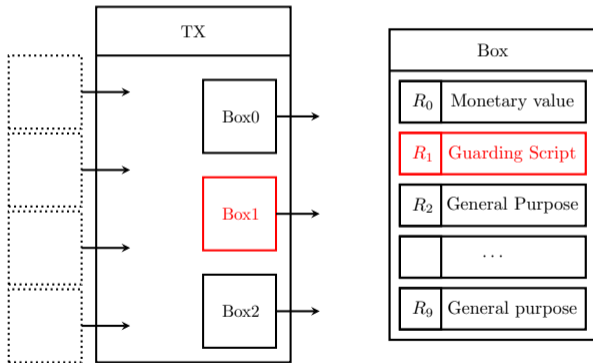
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## SCRIPTING LANGUAGE

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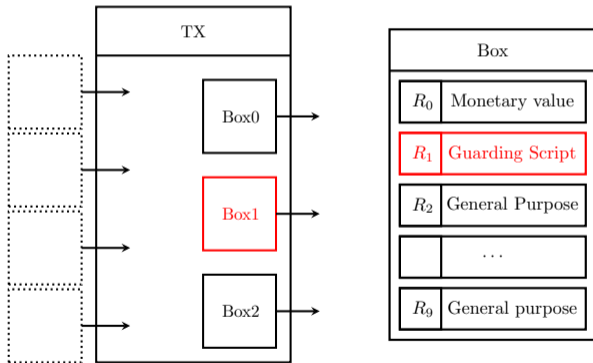
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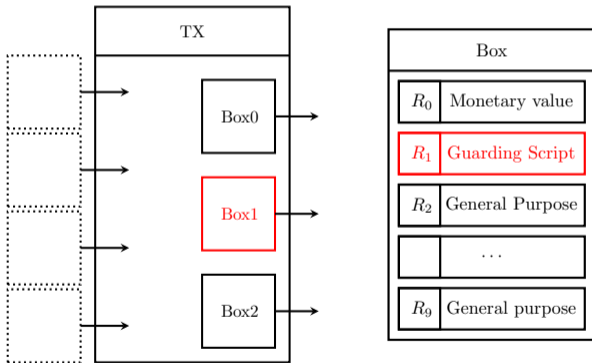
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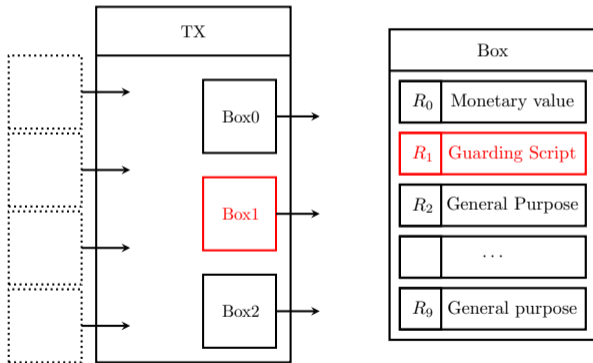
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  - SELF: Box
  - INPUTS: Array[Box]
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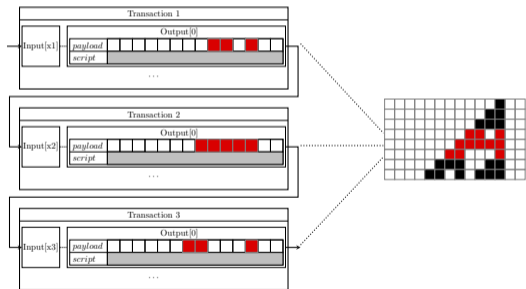
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```
let indices: Array[Int]=Array(0, 1, 2, 3, 4, 5)
let inLayer: Array[Byte]=SELF.R3[Array[Byte]].value
fun procCell(i: Int): Byte = {
  let l = inLayer((if(i==0) 5 else (i-1)))
  let c = inLayer(i)
  let r = inLayer((i + 1) % 6)
  ((1 * c * r + c * r + c + r) % 2).toByte
}
(OUTPUTS(0).R3[Array[Byte]].value==
  indices.map(procCell)) &&
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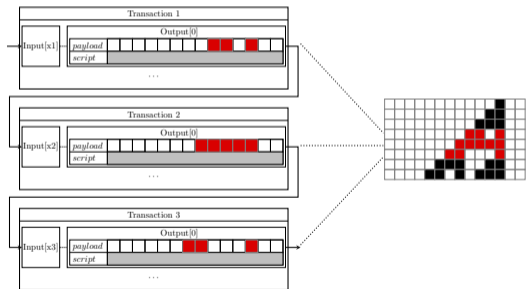
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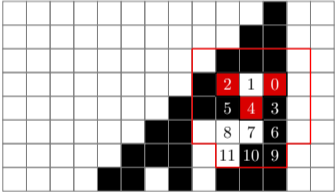
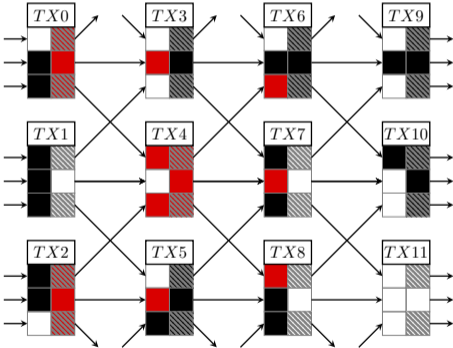
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We just allowed recursive calls by granting the script access to an output.

# RULE 110 CELLULAR AUTOMATON: INFINITE TAPE

Use transaction chaining to share memory between the transactions



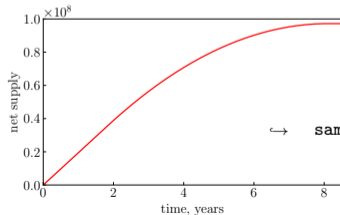
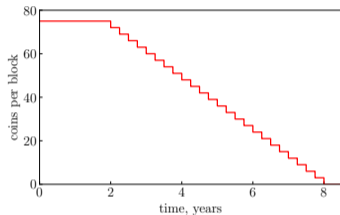
<https://git.io/vj6rX>

# EXAMPLES

- Crowdfunding
- Demurrage currency
- Oracles (w. authenticated state)
- Decentralized exchanges

`https://github.com/ergoplatform/ergo/blob/master/papers/yellow`

## PRACTICAL CASE: ERGO TOKEN EMISSION



```
let epoch = 1 + (HEIGHT-fixedRatePeriod)/epochLength
let out = OUTPUTS(0)
let coinsToIssue = if(HEIGHT < fixedRatePeriod) fixedRate
else fixedRate - oneEpochReduction*epoch
let correctCoinsConsumed = coinsToIssue==(SELF.value - out.value)
let sameScriptRule = SELF.propositionBytes==out.propositionBytes
let heightIncreased = HEIGHT>SELF.R3[Long].value
let heightCorrect = out.R3[Long].value==HEIGHT
let lastCoins = SELF.value<=oneEpochReduction
(correctCoinsConsumed && heightCorrect && heightIncreased &&
sameScriptRule) || (heightIncreased && lastCoins)
```

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# CONCLUSIONS

- Turing Completeness of the blockchain system can be achieved by unwinding the recursive calls between the transactions. It fully complies with the blockchain requirements, and does not require ad-hoc structures to bypass the halting problem
- We provide the explicit proof of the Turing completeness of Ergo blockchain scripting system. To our knowledge, this is the first proof of that kind
- The construction is explicit, and the functionality is fully implemented
- Self-reproducing coins allow one to make practical constructions. As an example, significant fraction of the validation rules can be brought from the hard-coded form to the scripting layer. Moreover, the logic of arbitrary complexity can be potentially implemented



**THANK YOU FOR YOUR ATTENTION**