A Unicorn Walks Into A Bar…

Or….

Accumulators for UTXOs

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Joint work with:

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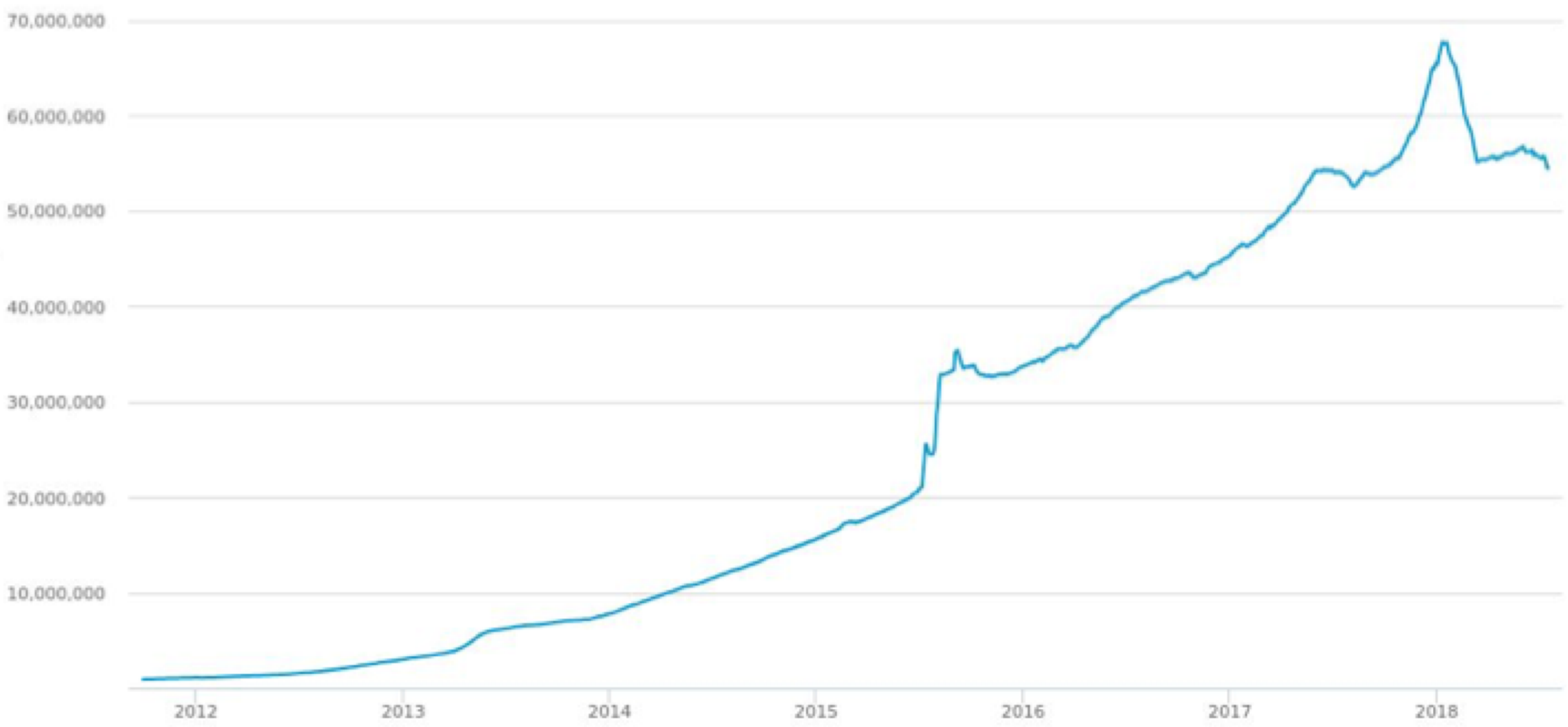
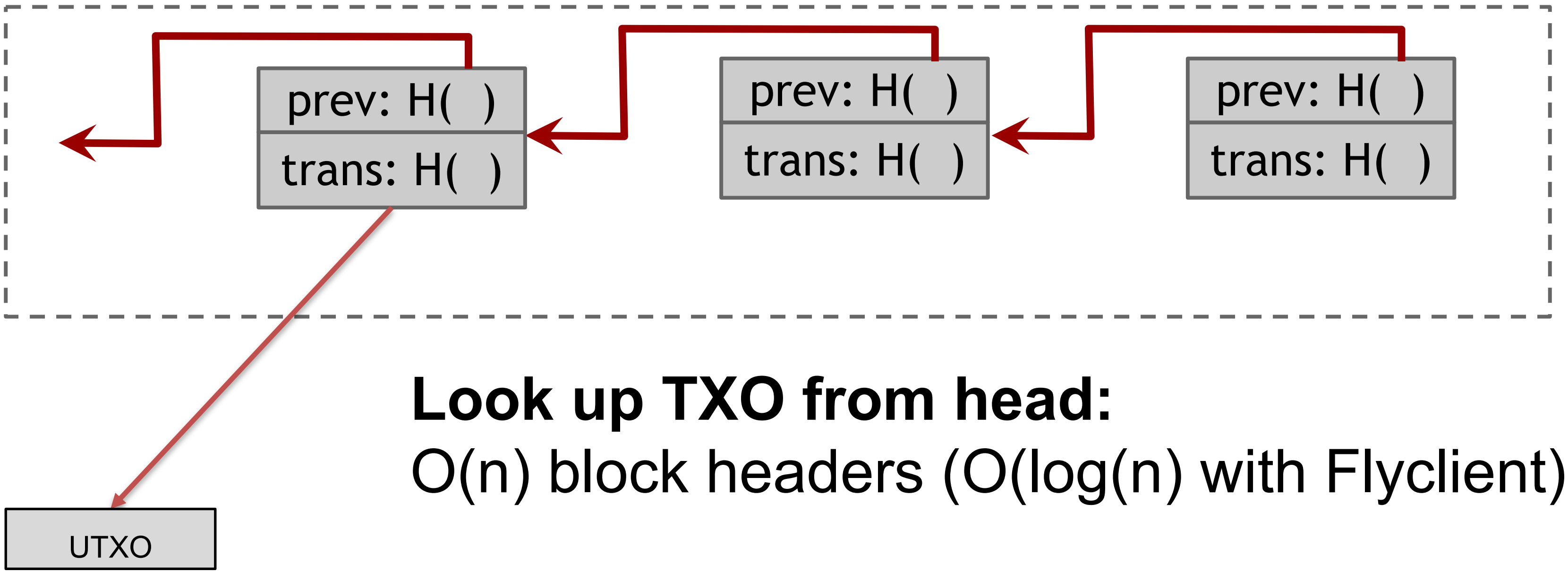
Stanford Blockchain Conference

- (Formerly BPASE)
- Conference: Jan. 30 - Feb 1, 2019.   (three days)
- Only technical submissions
- Submission: October 16th, 2018
- https://cyber.stanford.edu/sbc19
- Videos: https://tinyurl.com/bpasevideos
- Part of the Stanford Center for Blockchain Research (@CBRStanford)

UTXO Set: A Growing Problem



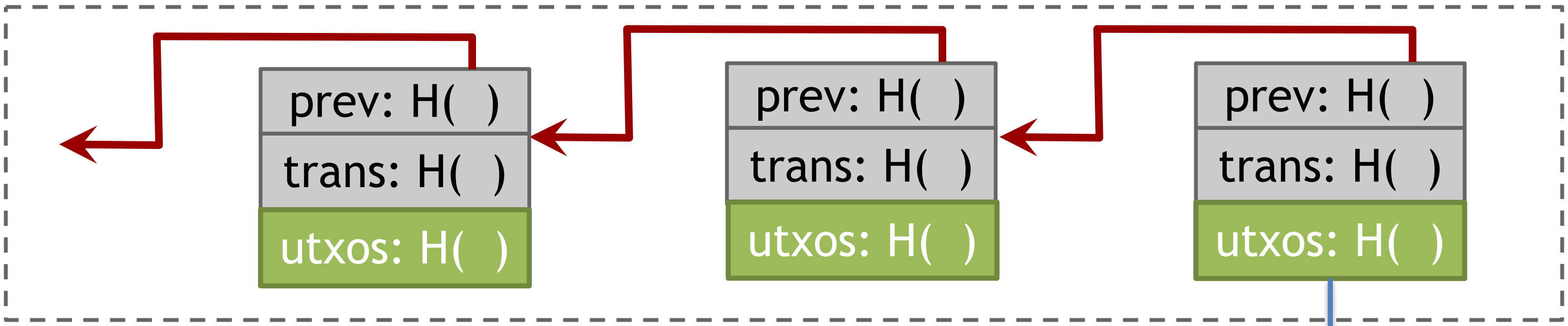
UTXOs



Look up TXO from head:
O(n) block headers (O(log(n) with Flyclient)

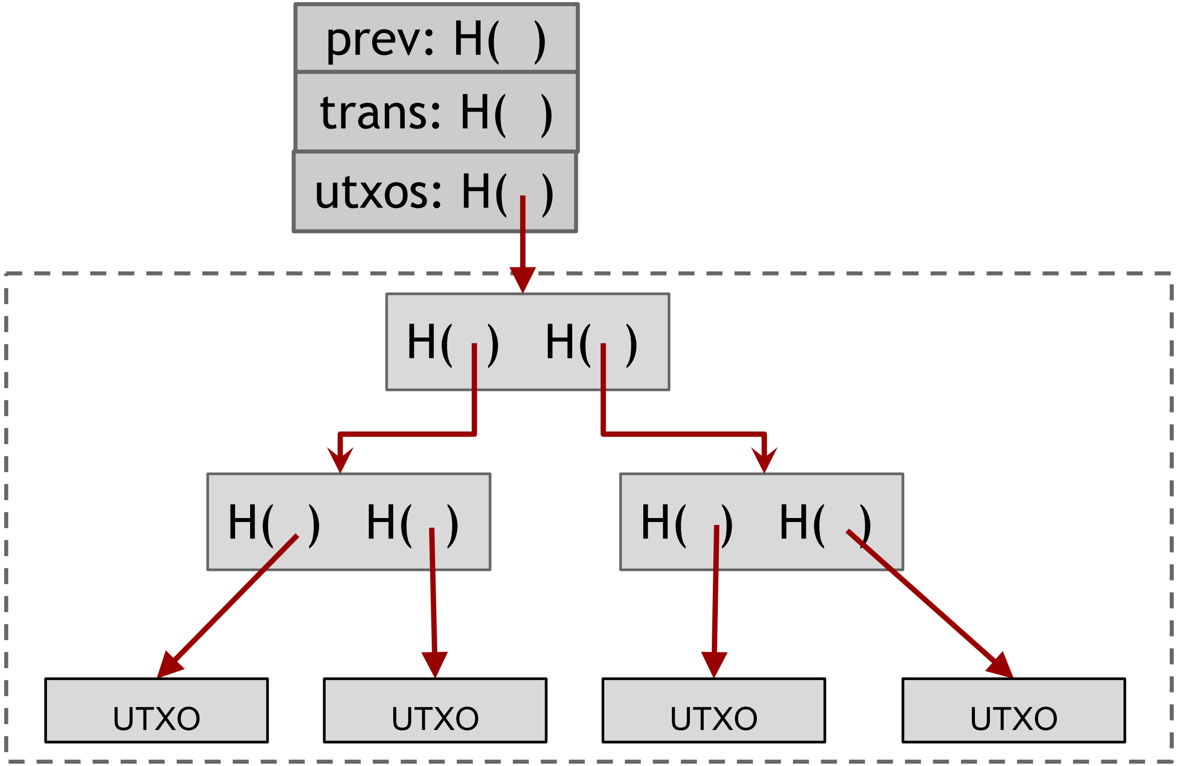
Look up UTXO: All transactions

UTXO Commitments [Miller,Todd,Dryja,...]



Consensus ensures:
All UTXO committed here

Merkle Trees



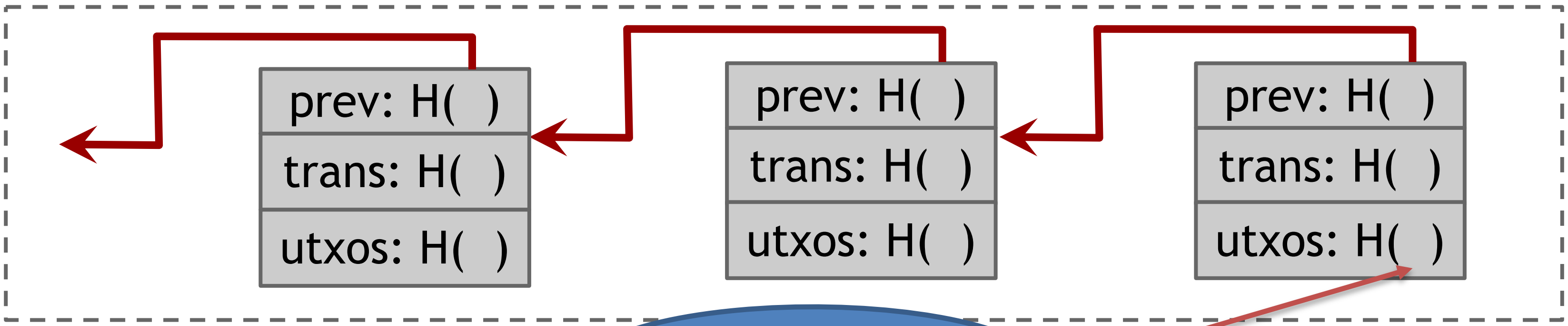
Inclusion: O(log(n))

Exclusion: O(log(n))[1]

Update: O(log(n))

[1] If sorted

Stateless Full Nodes/Mining



TX:
Spend UTXO 426
Proof: π

Problems with Merkle Trees

- Log(n) inclusion proof per transaction
- Inclusion proofs can hardly be aggregated
  - 600 GB naïvely
  - 160 GB with many optimizations
- Verification not that cheap
  - Full node sync too slow
  - Proposed for only old transactions

RSA Accumulators [CL02 …]

Setup:

- Choose N=pq where p, q are secret primes
- H: Hash function to primes in [0, 2^{λ}]
- $A_0 = g \in Z_N$ *(initial state)*

Add(A_i, x)

- $A_{i+1} = A_i^{H(x)}$

Del(A_i, x)

- $A_{i+1} = A_i^{1/H(x)}$

State after set S added:

$$u = \prod_{s \in S} s$$
$$A_t = g^u$$

Accumulator Proofs

InclusionProof(A,x):

- $\pi = A^{\frac{1}{x}} \in \mathbb{G}$
- Computed using trapdoor(p,q) Or $O(|S|)$

Verify$(\mathrm{A}, \mathrm{x}, \pi)$

- $\pi^x = A$

Efficient stateless updates: [LiLiXue07]

Exclusion(A, x)

- $A = g^u$
- $a \cdot x + b \cdot u = \mathrm{gcd}(x, u)$=1 See [LiLiXue07]

RSA = Trusted Setup?

N=p*q, p,q unknown

Efficient delete needs trapdoor

You can find Ns in the wild
(Ron Rivest Assumption)

Class Groups [BW88,L12]

CL(Δ) – Class group of quadratic number field $\mathbb{Q}(\sqrt{\Delta})$

$\Delta = -p$ (a large random prime)

Properties

- Element representation: integer pairs (a, b)

  $$|a| \approx |b| \approx \sqrt{-\Delta}$$

- Tasks believed to be hard to compute:

  Odd prime roots **Group order**

- $\Delta \approx 1536\ bits \Rightarrow$ 128 bit security

No trusted setup

RSA Accumulator State of Art

<u>Positives</u>

- <u>Constant</u> size inclusion proofs ($\approx$ 3000 bits)
  Better than Merkle tree for set size > 4000
- <u>Dynamic</u> stateless adds (can add elements w/o knowing set)
- Decentralized storage (no need for full node storage)
  - ➢ *Users maintain their own UTXOs and membership proofs*

<u>Room for improvement?</u> **This work**

- Aggregate/batch inclusion proofs (many at cost of one)
- Stateless deletes
- Faster (batch) verification

Aggregate Inclusion Proofs

$$\pi_1^x = A, \pi_2^y = A$$

$Shamir's\ Trick:$

$$a \cdot x + b \cdot y = 1$$

$$\pi_{1,2} = \pi_1^b \pi_2^a$$

$$\pi_{1,2}^{x \cdot y} = A$$

All inclusion proofs per block: 1.5kb

All inclusion proofs ever: 160GB -> 1.5kb

Stateless Deletion

Delete with trapdoor(A_t, x):

- $A_{t+1} = A_t^{\frac{1}{x}}$

Using knowledge of p, q

Delete with inclusion proof($\mathrm{A_t, x}, \pi$)

- $A_{t+1} = \pi$;

$\pi = \mathrm{g}^{\frac{u}{x}}$

BatchDelete($\mathrm{A_t, x, y}, \pi_1, \pi_2$)

- Compute $\pi_{1,2}$ s.t. $\pi_{1,2}^{x \cdot \mathrm{y}} = A_t$
- $A_{t+1} = \pi_{1,2}$

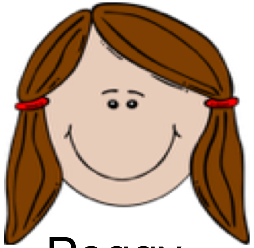
No State,
no Trapdoor,
asynchronous

Too slow?

- Openssl 2048 bit RSA:
  - 219 updates per second
  - Verification/Full sync would be problematic
- Class groups: No good benchmarks yet

Wesolowski Proof [Wesolowski'18]

$(x, y, T): x^{2^T} = y$



Peggy

y →

← Random λ bit prime l

Victor

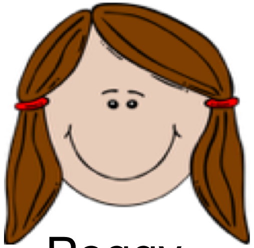
Computes
q,r s.t.
$2^T = q \cdot l + r$ and
$0 \leq r < l$

$\pi = x^q$ →

Computes
$r = 2^T \bmod l$
Checks:
$\pi^l x^r = y$
$x^{q \cdot l} x^r = x^{2^T}$

Proof of Exponentiation

$(x, y, \alpha): x^{\alpha} = y$



Peggy

$\xrightarrow{\quad y \quad}$

$\xleftarrow{\text{Random } \lambda \text{ bit prime } l}$

Victor

Computes
q,r s.t.
$\alpha = q \cdot l + r$ and $0 \leq r < l$

$\xrightarrow{\quad \pi = x^{q} \quad}$

Computes
$r = \alpha \bmod l$
Checks:

$$\pi^{l} x^{r} = y$$
$$x^{q \cdot l} x^{r} = x^{\alpha}$$

Proof of Exponentiation Efficiency

$(x, y, \alpha): x^{\alpha} = y$

Direct Verification:
$x^{\alpha} = y \in \mathbb{G}$

PoE Verify:
$r = \alpha \bmod l$
$\pi^{l} g^{r}$

Exponentiation in $\mathbb{G}$ vs. 128 bit long-division:
5000x difference for 128 bit security

Fast Block Verification

Header:
TXs: Spent s, new N
BLS σ
$A_{t+\frac{1}{2}}, A_{t+1}, PoE$

A_t → Remove s → $A_{t+\frac{1}{2}}$

$A_{t+\frac{1}{2}}$ → Add N → A_{t+1}

Verify σ

Verify PoE($A_{t+\frac{1}{2}}^{\prod_{s\in S} s} = \mathrm{A_t}$)

PoE($A_{t+\frac{1}{2}}^{\prod_{n\in N} N} = \mathrm{A_{t+1}}$)

Performance

Macbook, Java BigInteger, JDK Hash

Merkle Tree: 26 x SHA-256:
8.5 μs

Add: g^x mod N, |x|=256 bit |N|=3072:
1535 μs

Verify: x mod l, |x|=256 bit |l|=128 bit
0.3 μs

Classgroups?

Vector Commitments

VC=Commit($a_1, a_2, a_3, \dots, a_n$)

Merkle trees are VCs not just accums!

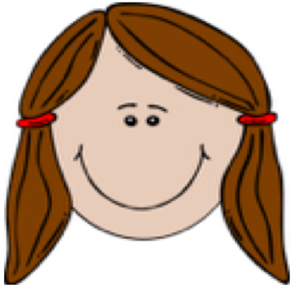
π =Open(VC,a_i, i)

Verify(VC, π, a_i, i) $= \{0,1\}$

Classical VCs: Verifier requires GBs of memory

New VCs: Zero-memory

Short IOPs (STARKs etc.)

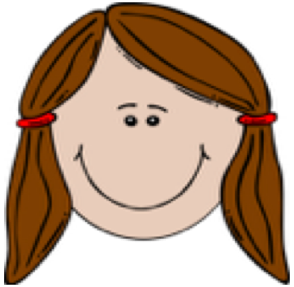


MT=Commit(Long Proof)

$i_1, \ldots, i_\lambda$

$\pi_{i_1}, \ldots, \pi_{i_\lambda}$ and Merkle Paths

Short IOPs (STARKs etc.)



VC=Commit(Long Proof)

$i_1, \dots, i_\lambda$

$\pi_{i_1}, \dots, \pi_{i_\lambda}$ and 1 VC Opening

200kb
vs.
600kb

A Unicorn Walks Into A Bar…

***A**ccumulators, **Uni**ons, **W**esolowski, **I**OPs, **A**ggregation and **B**lockchains*

References

- CL02: Camenisch Lysanskaya 2002 Dynamic Accumulators
- LiLiXue07: Li, Li, Xue 2007 Universal Accumulators
- CF: Catalone Fiore: Vector Commitments
- Todd: https://petertodd.org/2016/delayed-txo-commitments#further-work
- MMR: https://github.com/opentimestamps/opentimestamps-server/blob/master/doc/merkle-mountain-range.md
- UTXO: https://bitcointalk.org/index.php?topic=101734.0
- BW88: Buchmann and Williams